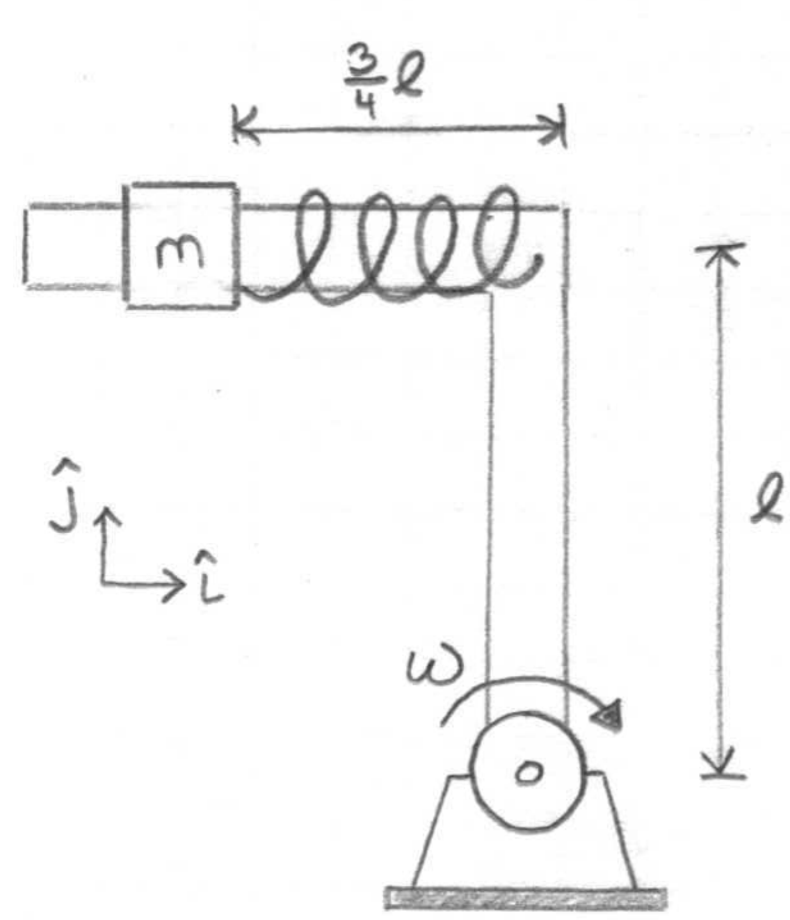
13.29



Given: $\ell = 0.4\,m$, $\omega = 2\,rad/s$, $k = 6\,N/m$

$m = 0.5\,kg$, Find ℓ_0

$R = \sqrt{0.4^2 + 0.3^2} = 0.5$

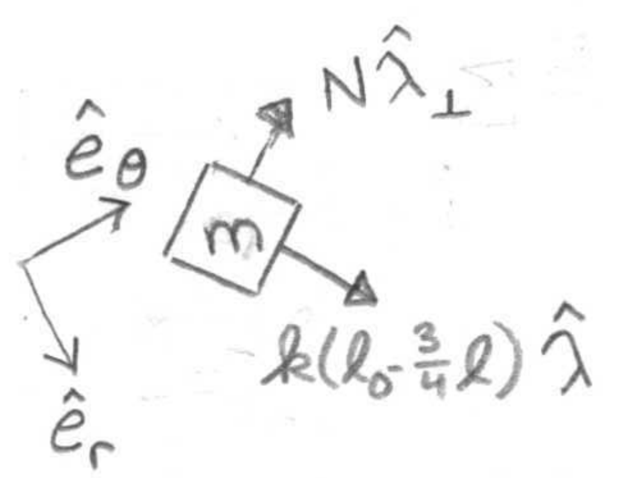
For a circular motion,

$\vec{a} = -R\dot{\theta}^2 \hat{e}_r + R\ddot{\theta}\hat{e}_\theta$, where

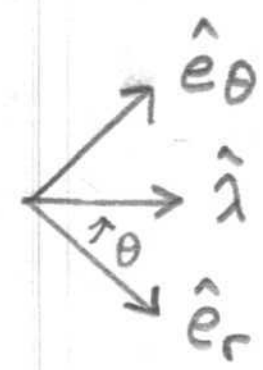
$\dot{\theta} = \omega$ and $\ddot{\theta} = 0$

$\Rightarrow \vec{a} = -R\dot{\theta}^2 \hat{e}_r$

FBD of mass:



Find $\hat{\lambda}$:



$\hat{\lambda} = \frac{3}{5}\hat{e}_r + \frac{4}{5}\hat{e}_\theta$

$\sum \vec{F} = m\vec{a} \rightarrow N\hat{\lambda}_\perp + k(\ell_0 - \frac{3}{4}\ell)\hat{\lambda} = -mR\omega^2 \hat{e}_r$

$\{\}\cdot\hat{\lambda} \rightarrow k(\ell_0 - \frac{3}{4}\ell) = -mR\omega^2\left(\hat{e}_r \cdot \left(\frac{3}{5}\hat{e}_r + \frac{4}{5}\hat{e}_\theta\right)\right)$

OR $k(\ell_0 - \frac{3}{4}\ell) = -\frac{3}{5}mR\omega^2$

$\ell_0 - \frac{3}{4}\ell = \frac{-3mR\omega^2}{5k}$

$\therefore \ell_0 = \frac{3}{4}\ell - \frac{3}{5k}mR\omega^2 = \frac{3}{4}(0.4\,m) - \frac{3}{5(6\,N/m)}(0.5\,kg)(.5\,m)(2\,rad/s)^2$

$= 0.3 - 0.1$

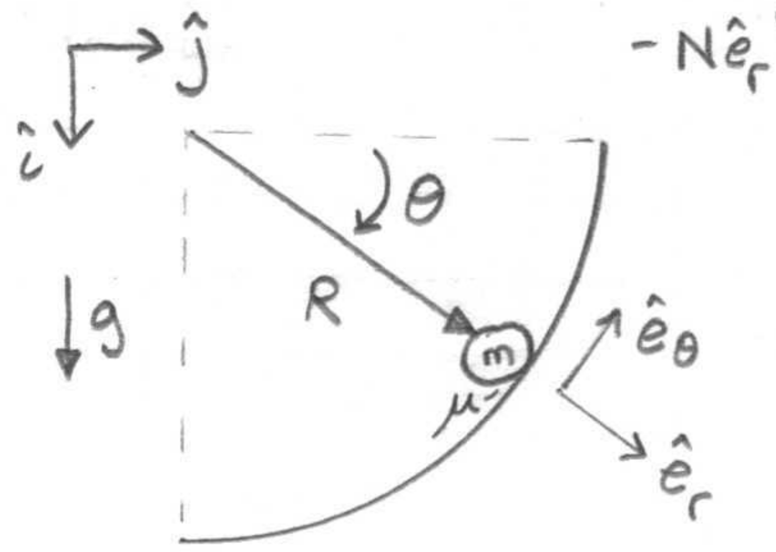
$\therefore \boxed{\ell_0 = 0.2\,m}$

13.38

COVERED IN LECTURE ON 4/10. SEE ATTACHED.
(P.

13.43

a) FBD:



$$\Sigma\vec{F} = m\vec{a}$$

$$\{-N\hat{e}_r - \mu N\hat{e}_\theta - mg\hat{j} = mR(-\dot{\theta}^2\hat{e}_r + \ddot{\theta}\hat{e}_\theta)\}$$

$$\{\}\cdot(-\hat{e}_\theta + \mu\hat{e}_r) \rightarrow -\mu N + \mu N - \mu mg\sin\theta$$
$$+ mg\cos\theta = mR(-\mu\dot{\theta}^2 - \ddot{\theta})$$

$$\therefore \cancel{m}R\mu\dot{\theta}^2 + \cancel{m}R\ddot{\theta} = \mu\cancel{m}g\sin\theta - \cancel{m}g\cos\theta$$

$$\text{OR} \quad \boxed{\ddot{\theta} + \mu\dot{\theta}^2 = \frac{g}{R}(\mu\sin\theta - \cos\theta)}$$

b) Given: $R = 0.5\,m$, $m = 0.1\,g$, $g = 10\,m/s^2$, $\mu = 0.2$

$\theta_0 = 0$, $v_0 = 10\,m/s = R\dot{\theta}_0 \quad \therefore \quad \dot{\theta}_0 = \frac{10}{.5} = 20\,rad/s$

Separate into two 1^{st} order: (let $\omega = \dot{\theta}$)

$$\therefore \quad \dot{\theta} = \omega$$
$$\dot{\omega} + \mu\omega^2 = \frac{g}{R}(\mu\sin\theta - \cos\theta)$$
$$\rightarrow \quad \dot{\omega} = \frac{g}{R}(\mu\sin\theta - \cos\theta) - \mu\omega^2$$

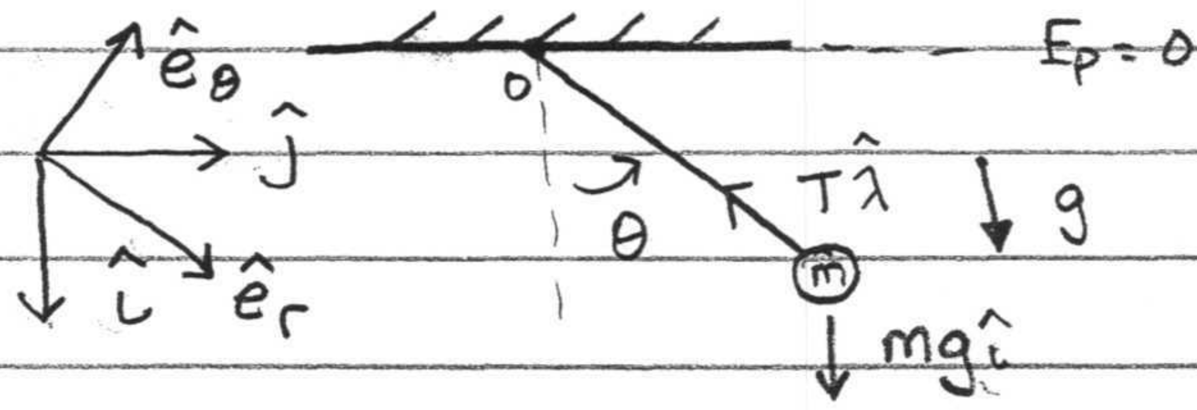
See attached code on next page.

The velocity at the bottom of the chute, from Matlab, is <u>6.94 m/s</u>

```matlab
function Prob1343()
% Problem 13.43 Solution
% April 17, 2008

% VARIABLES
R= 0.5; % radius [m]
m= .1/1000; % mass [kg]
g= 10; % gravity accel. [m/s^2]
mu= .2; % friction coefficient

% Initial Conditions
th0= 0; % theta at t=0
om0= 20; % omega (theta dot) at t=0

z0= [th0; om0]; % pack variables

tspan= [0 10];

options = odeset('events', @stopevent);

[t zarray]= ode45(@rhs,tspan,z0,options,R,m,g,mu);

% Unpack variables
th= zarray(:,1);
om= zarray(:,2);

plot(R*cos(th),-R*sin(th));

fprintf('Velocity at bottom of chute is %f m/s\n',R*om(end));

% ANSWER: Velocity at bottom of chute is 6.941249 m/s %

end

% THE DIFFERENTIAL EQUATIONS (RIGHT HAND SIDE)
function zdot = rhs(t,z,R,m,g,mu)

% Unpack variables
th= z(1);
om= z(2);

% The equations
thdot= om;
omdot= g/R*(mu*sin(th)-cos(th))-mu*om^2;

% Pack the rate of change variables
zdot= [thdot; omdot];
end

% STOP EVENT CRITERIA
function [value, isterminal, dir] = stopevent(t,z,R,m,g,mu)
th= z(1);
om= z(2);
value= th-pi/2;
isterminal= 1;
dir= +1;
end
```

SIMPLE PENDULUM

Derive governing equations in 5 ways:



note: $\begin{Bmatrix} \hat{\imath} \\ \hat{j} \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \hat{e}_r \\ \hat{e}_\theta \end{Bmatrix}$

1) LMB IN RECT. COORDINATES

$\sum \vec{F} = m\vec{a} \therefore \quad mg\hat{\imath} - T\hat{\lambda} = m(\ddot{x}\hat{\imath} + \ddot{y}\hat{j})$

$\{\} \cdot \hat{\imath} \rightarrow \quad mg - T\cos\theta = m\ddot{x} \quad (1)$

$\{\} \cdot \hat{j} \rightarrow \quad -T\sin\theta = m\ddot{y} \quad (2)$

We know $x^2 + y^2 = L^2$ (3) - diff. twice w/chain rule

Also substitute $\cos\theta = x/L$, $\sin\theta = y/L$

$$\therefore \boxed{\begin{aligned} \ddot{x} &= \frac{1}{x^2+y^2}\left[gy^2 - (\dot{x}^2 + \dot{y}^2)x\right] \\ \ddot{y} &= \frac{1}{x^2+y^2}\left[y(-gx + \dot{x}^2 + \dot{y}^2)\right] \\ T &= \frac{1}{x^2+y^2}\left[Lgmx + mL(\dot{x}^2 + \dot{y}^2)\right] \end{aligned}}$$

Then, we want $\theta(t) = \tan^{-1}\left[y(t)/x(t)\right]$

↳ but we get errors when $\theta = \pi/2$ (i.e. $x = 0$)

2) LMB IN POLAR COORDINATES

$\sum \vec{F} = m\vec{a} \therefore \quad mg\hat{\imath} - T\hat{e}_r = m(-L\dot{\theta}^2\hat{e}_r + L\ddot{\theta}\hat{e}_\theta)$

$\{\} \cdot \hat{e}_\theta \rightarrow \quad \cancel{m}g(-\sin\theta) = \cancel{m}L\ddot{\theta} \quad (1)$

$\{\} \cdot \hat{e}_r \rightarrow \quad mg\cos\theta - T = -mL\dot{\theta}^2$ (to find T)

Using (1), $\boxed{\ddot{\theta} + \frac{g}{L}\sin\theta = 0}$ - eqn. of motion! (*)

... CONTINUED

3) ANGULAR MOMENTUM BALANCE (about O)

$$\Sigma \vec{M}_o = \dot{\vec{H}}_o, \quad \vec{r}_{m/o} \times m\vec{v} = \vec{H}_o$$
$$= \vec{r}_{m/o} \times m\vec{a} \quad \text{- now tension is gone}$$

$$\vec{r}_{m/o} \times mg\hat{\imath} = \vec{r}_{m/o} \times m\vec{a} = L\hat{e}_r \times m(-L\dot{\theta}^2\hat{e}_r + L\ddot{\theta}\hat{e}_\theta)$$

$$\therefore \quad Lmg(\hat{e}_r \times \hat{\imath}) = -mL^2\dot{\theta}^2(\hat{e}_r \times \hat{e}_r) + mL^2\ddot{\theta}(\hat{e}_r \times \hat{e}_\theta)$$
$$\cancel{L}\cancel{m}g(-\sin\theta)\hat{k} = 0\hat{k} + \cancel{m}L^{\cancel{2}}\ddot{\theta}\hat{k}$$
$$\{\}\cdot\hat{k} \rightarrow -mg\sin\theta = mL\ddot{\theta}$$

$$\therefore \boxed{\ddot{\theta} + \frac{g}{L}\sin\theta = 0} \quad (*)$$

4) ENERGY BALANCE

$$\dot{E}_k = P \text{ (power)} \rightarrow P = \vec{F}\cdot\vec{v}, \; \dot{E}_k = \frac{d}{dt}\left(\frac{1}{2}m\vec{v}\cdot\vec{v}\right)$$

$$\therefore \quad \frac{1}{2}m\frac{d}{dt}(\vec{v}\cdot\vec{v}) = \vec{F}\cdot\vec{v}$$
$$\frac{1}{2}m\frac{d}{dt}(L^2\dot{\theta}^2) = (-T\hat{e}_r + mg\hat{\imath})\cdot(L\dot{\theta}\hat{e}_\theta)^*$$

$$^* \; \vec{r} = r\hat{e}_r, \text{ so } \dot{\vec{r}} = r\dot{\hat{e}}_r = r(\dot{\theta}\hat{e}_\theta)$$

$$\therefore \quad \frac{1}{\cancel{2}}\cancel{m}L^{\cancel{2}}(\cancel{2}\dot{\theta}\ddot{\theta}) = mgL\dot{\theta}(\hat{\imath}\cdot\hat{e}_\theta) = \cancel{m}g\cancel{L}\cancel{\dot{\theta}}(-\sin\theta)$$
$$L\ddot{\theta} = -g\sin\theta \rightarrow \ddot{\theta} + \frac{g}{L}\sin\theta = 0 \quad ✓$$

5) CONSERVATION OF ENERGY

For a conservative system $E_T = E_k + E_p = k$ OR $\dot{E}_k + \dot{E}_p = 0$

$\Rightarrow E_k = \frac{1}{2}mL^2\dot{\theta}^2, \quad E_p = -mgL\cos\theta$

$$\therefore \cancel{m}L^{\cancel{2}}\cancel{\dot{\theta}}\ddot{\theta} + \cancel{m}g\cancel{L}\cancel{\dot{\theta}}\sin\theta = 0 \quad \text{OR} \quad \ddot{\theta} + \frac{g}{L}\sin\theta = 0 \; ✓$$

To integrate, assume small angles: $\ddot{\theta} + \frac{g}{L}\theta = 0$ and solve.